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# Comparison of Interpolation Methods as Applied to Time Synchronous Averaging

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# TO TIME SYNCHRONOUS AVERAGING

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## ABSTRACT

Several interpolation techniques were investigated to determine their effect on time synchronous averaging of gear vibration signals and also the effects on standard health monitoring diagnostic parameters. The data was also digitally resampled to determine the effect of lower acquisition rates. The analysis used previously recorded vibration data taken during Health and Usage Monitoring gear testing at the NASA Glenn Research Center. The gear testing monitored the development of surface pitting fatigue on aerospace quality spur gears. Linear, cubic and spline interpolation methods were investigated. Comparisons between the resultant averages show that while there are differences in the resultant time synchronous averages, the differences are not obvious. The diagnostic parameters tested were FM4 and NA4. There are significant differences in the percent deviation curves which imply that the magnitudes of the errors increase as the sample rate decreases.

## INTRODUCTION

One of the most important areas of research in rotorcraft drivetrains is in fault detection. The detection of gear fatigue faults is critical to safe, on-condition maintenance. The current method of maintenance of rotorcraft is based on number of flight hours, not the amount of wear that occurs. Therefore, there are potentially many hours of usable life thrown away during transmission overhauls. An effective health monitoring system would allow the part to be more completely used.

One of the fundamental components to a drivetrain monitoring system is the ability to time synchronous average the vibration signal. In time synchronous averaging, the vibration is averaged during many successive revolutions of the gear being examined. This provides a necessary filter to allow the repetitive gear signals to dominate over the non-synchronous background noise of the rest of the components. Since the vibration signal is comprised of

discrete points (which most likely do not represent the same point in the rotation of the gear); the signal must be resampled into a fixed number of points per revolution. Typically, the number of points chosen is a factor of two for ease in later signal processing techniques. It is during this resampling that an interpolation method is used to match the exact points in a revolution for all revolutions.

There are many different interpolation techniques available. Time synchronous averaging places two constraints on which techniques to use: accuracy and computation time. Obviously, the resultant signal is only as accurate as the interpolation technique used. Typically the more accurate the interpolation, the longer that it takes to perform the interpolation. The more computation time that is required, the longer it takes to get an indication of damage from the detection methods. It is because of these somewhat opposing goals that this research was performed. It is the subject of this paper to determine if there are significant differences between interpolation techniques to warrant the additional computational resources required.

## DESCRIPTION OF INTERPOLATION METHODS

This paper discusses an investigation comparing three interpolation techniques (linear, cubic and spline). There are many more techniques available, but typically, they require more computer resources. The three time synchronous averages are compared directly as well as their effects on a couple of standard gear diagnostic parameters. The data analyzed is based on previously digitized vibration data recorded from a surface fatigue gear test. Although just a sample from the entire run could have been used, all of the data samples were used to determine that there was no effects based on the level of damage to the gear.

The first technique investigated is linear interpolation. Although it is the most common (and fastest), typically it is the least accurate. It is also the simplest form of interpolation. The technique simply connects two data points with a straight line, and uses the equation of that line segment to determine the desired value. For many instances, linear interpolation is sufficient.

The second technique investigated was cubic interpolation. This technique is the second fastest technique investigated. This technique curve fits a cubic equation to the provided data, and then solves the equation for the point desired. Depending upon the accuracy of the curve fit and the data, this could either be more or less accurate than linear interpolation.

The third technique investigated is cubic spline interpolation. In this technique a series of functions are used to determine the interpolated data points. There is a separate function between each set of data points. At each interval, the equation has continuity through the requirement of matching the first and second derivatives of the preceding and following functions. This technique was significantly slower than the others were. Although the slowest, typically it is the most accurate of those that were investigated. Since the calculation is based on value and derivatives at each of the surrounding points, the estimate should be the most accurate.

## OVERVIEW OF DETECTION METHODS

Although the effects of different interpolation techniques can be directly compared based on the time synchronous averages, it is important to determine if there are any significant effects on the standard gear vibration diagnostic parameters. Many of the most common gear diagnostic parameters were tested using the time synchronous averages from the three interpolation schemes. The linear interpolation is taken to be the baseline for comparison. The following parameters were investigated: FM4 and NA4. A small description of each of these methods is provided below. Details of these parameters are discussed in the references.

The FM4 parameter, developed by Stewart [1], is the normalized fourth statistical moment of a residual vibration signal. The meshing frequency and its sidebands are removed from the signal and this becomes the residual vibration signal. The moment is taken, and normalized by the square of the variance. Under certain circumstances, the variance will increase, causing the parameter to decrease as damage progresses. FM4 is dimensionless and has a value of three under normal conditions.

The NA4 parameter developed by Zakrajsek [2, 3] is the similar to FM4, with the following two exceptions. First, the residual signal retains the first sidebands. Second, the parameter is normalized by the square of the time-averaged variance as opposed to the current variance. This parameter can also decrease as damage progresses, although to a lesser extent as compared to FM4. NA4 is also dimensionless, with a value of three under normal conditions.

## APPARATUS AND GEAR DAMAGE REVIEW

The testing was completed using the gear fatigue test rigs at the NASA Glenn Research Center. The rigs are used to study the effects of gear tooth design, gear materials and lubrication on the fatigue lives of aerospace quality gears. In addition, health-monitoring research is performed both on- and off-line. Vibration data from accelerometers mounted on the rig as well as a tachometer pulse was captured with a personal computer using an analog to digital conversion board and an anti-aliasing filter. The gears have 28 teeth, a pitch diameter of 88.9 mm (3.50 in), and a face width of 6.35 mm (0.25 in). The gear faces are run offset to achieve a 2.79 mm (0.11 in) contact width to allow a higher surface stress without a corresponding increase in the bending stress. The test pair transmit 143 kW (192 hp) at 10,000 revolutions per minute.

A sample test run was used to determine the effect of sampling rates and interpolation methods on vibration health monitoring parameters. For this case, the testing was stopped after 337 hours when the RMS vibration limit was exceeded. The gears exhibited significant damage during the test. Only 11 of the 56 teeth (28 on each gear) had no damage. Light to moderate (depth <0.5mm) pitting occurred on 40 of the teeth. Heavy pitting occurred on 11 of the teeth. (All heavy pitting was also accompanied by light to moderate pitting.

# DISCUSSION OF RESULTS

The vibration data was post-processed on a personal computer at acquisition rates of 125,000, 62,500 and 31,250 samples per second. The data was originally sampled at 125,000 samples per second during the test and saved on disk. The resampling at 62,500 and 32,500 samples per second was accomplished by simply taking ever other and every fourth point respectively from the original data.

The time synchronous averages for two different examples in the run are presented. Figure 1 presents the time synchronous average at the beginning of the test, while Figure 2 presents the time synchronous average at 256.8 hours, a point with known significant damage. The different rows reflect the different interpolation methods beginning with the linear method and proceeding to the cubic method and finally the cubic spline method. The different columns represent the different acquisition rates starting at left with the 125,000 Hz rate going to 62,500 Hz and then ending with the 31,250 Hz acquisition rate. Both figures illustrate how details in a signal will be rounded over with decreasing acquisition rates. There are no visible differences between the cases where only the interpolation method was changed. The most prominent effects are due to data acquisition rates.

Close inspection between Figures 1 and 2 indicate that not only has the amplitude of the signal has increased, but certain peaks are missing, amplified, or just changed. It takes some fine examination, but it is clear that something significant has happened to whatever is creating this signal.



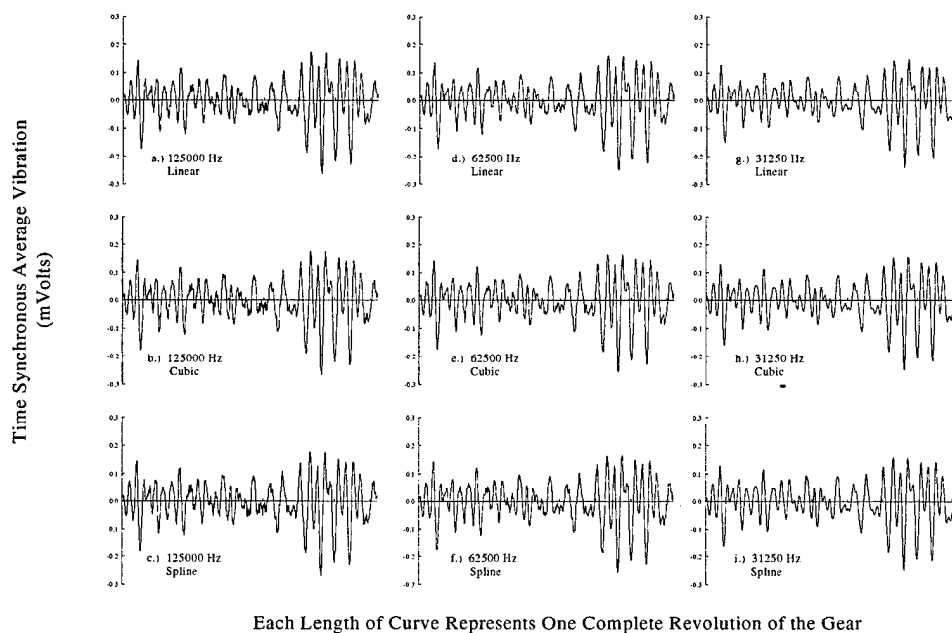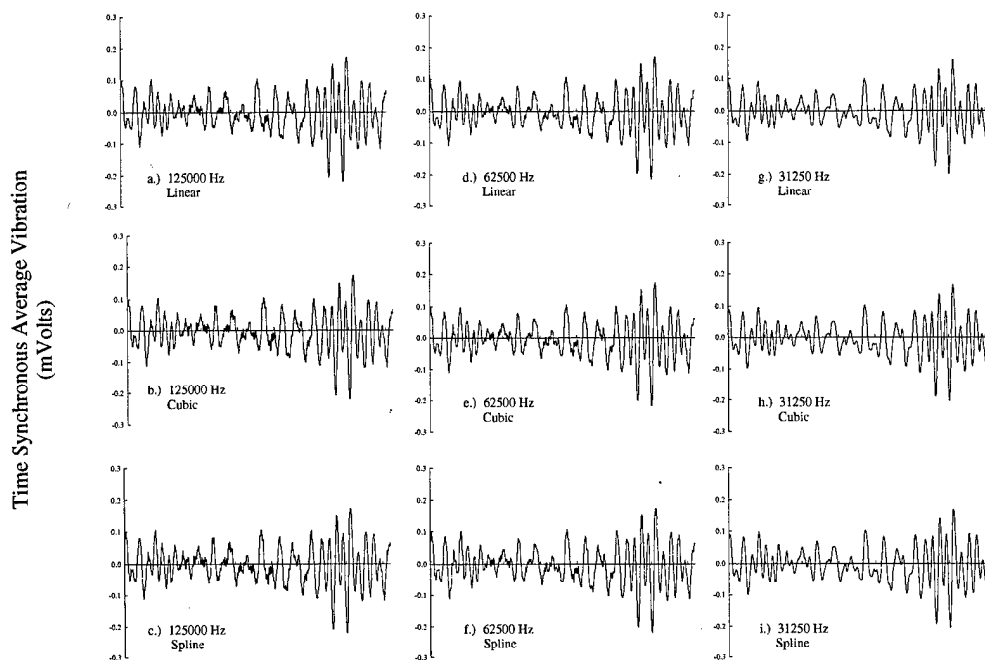Each Length of Curve Represents One Complete Revolution of the Gear

Figure 1. Time Synchronous Average at beginning of run.

Each Length of Curve Represents One Complete Revolution of the Gear

Figure 2. Time Synchronous Average at 256 hours run time.

One of the important considerations in the choice of an interpolating routine is the length of time that it takes to execute. Table I shows the results of some benchmark testing using the programs developed for this report. It is more important to look at the relative time differences. Given the performance of the spline interpolation method both in speed and in accuracy, it most likely will not be the preferred.

Table I. Interpolation Execution Times.

| Method | Average Execution Time (sec) | Normalized Execution Time |
|--------|------------------------------|---------------------------|
| Linear | 0.328 | 1 |
| Cubic | 0.397 | 1.21 |
| Spline | 1.718 | 5.23 |

Figure 3 shows the percent deviation from the 125,000 Hz linear interpolation case for FM4. Although there may appear to be only one curve on a figure, there are two on 3a, and three on 3b, and 3c. FM4 appears to be less sensitive (comparatively speaking) to both acquisition rate and interpolation technique. There appears to be no significant differences between the three interpolation methods. The data acquisition rate has the most significant effect. This results in an increase in the variance of the curve as the sampling rate decreases. In this particular configuration, there is no sufficient reason to chose either cubic or cubic spline interpolation over linear. As with the other cases, the highest data acquisition rate is desired.
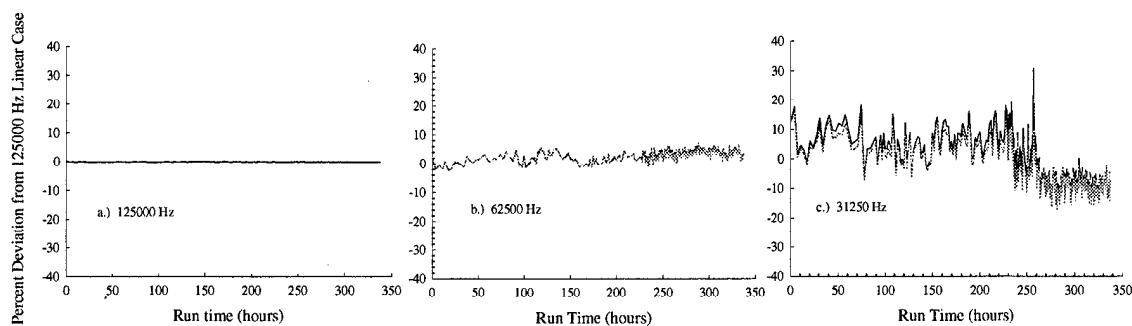
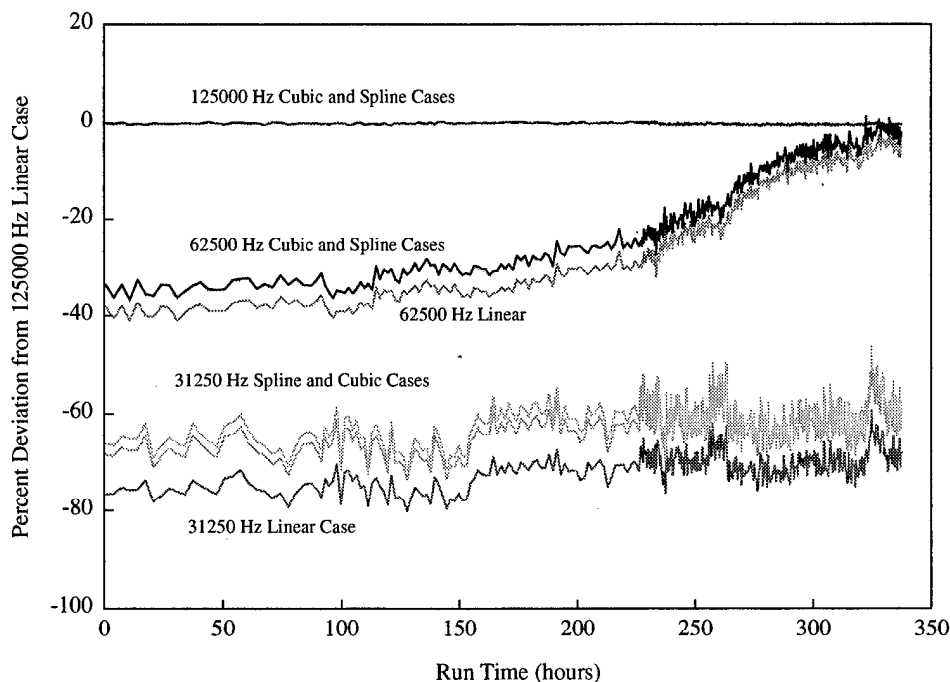Figure 3. Percent Deviation Curves for FM4.



Figure 4. Percent Deviation Curves for NA4.

Figure 4 shows the percent deviation from the 125,000 Hz linear case for NA4. Several interesting ideas become apparent. The NA4 parameter appears to be very sensitive to both sampling rate and to interpolation methods. As the data acquisition rate decreases, the deviation from the 125,000 Hz linear case increases. As the acquisition rate decreases the differences between interpolation methods increases, as well as the variance of the difference curve. The cubic and spline methods tend to produce values closer to the 125,000 Hz baseline than the corresponding linear interpolation. In this particular configuration, there is no apparent reason to choose cubic or spline interpolation for the 125,000 Hz sample rate. Overall, the variations are extreme: up to 78% deviations.

## SUMMARY AND CONCLUSIONS

This report examined the effect of two items on the time synchronous average: interpolation method and data acquisition rate. The report focussed on the results of post analysis of actual gear fatigue vibration data. The time synchronous average was calculated using the linear, cubic, and cubic spline methods. The original data was sampled at 125,000 samples per second. It was then resampled at both 62,500 and 31,250 samples per second. This data was then analyzed using the FM4 and NA4 parameters. The actual parameter values were not compared. Their differences from a baseline (125,000 Hz and linear interpolation methods) were compared. The following conclusions can be made:

1.  The accuracy of the time synchronous average decreased as the data acquisition rate decreased. Small peaks and valleys get rounded over as the rate decreases. (In this particular case, 62,500 samples per second might suffice, but 125,000 samples per second is really desirable.)

2.  There was no significant difference in execution time between the linear and cubic interpolation methods. The cubic spline method required significantly more time and did not seem to have an appropriate increase in accuracy.

3.  The data acquisition rate had the most significant effect on the parameter deviation. As the rate decreased, the deviation increased. In general, the cubic and spline gave the minimum deviation. Due to the difference in execution times, if linear proves insufficient, use the cubic interpolation method. The higher the acquisition rate, the less variance among the interpolation methods.

4.  The NA4 parameter seemed significantly more sensitive to both data acquisition rate as well as interpolation method when compared to the FM4 parameter.

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